Two-Loop Corrections in the MSSM with Complex Parametes

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1. Introduction and motivation

2. Two-loop corrections in the cMSSM Higgs sector

3. Numerical results

4. Conclusions

1. Introduction and motivation

Enlarged Higgs sector: Two Higgs doublets

$$H_1 = \begin{pmatrix} H_1^1 \\ H_1^2 \end{pmatrix} = \begin{pmatrix} v_1 + (\phi_1 - i\chi_1)/\sqrt{2} \\ -\phi_1^- \end{pmatrix}$$

$$H_2 = \begin{pmatrix} H_2^1 \\ H_2^2 \end{pmatrix} = e^{i\xi} \begin{pmatrix} \phi_2^+ \\ v_2 + (\phi_2 + i\chi_2)/\sqrt{2} \end{pmatrix}$$

$$V = m_1^2 H_1 \bar{H}_1 + m_2^2 H_2 \bar{H}_2 - (m_{12}^2 \epsilon_{ab} H_1^a H_2^b + \text{h.c.}) + \underbrace{\frac{g'^2 + g^2}{8}} (H_1 \bar{H}_1 - H_2 \bar{H}_2)^2 + \underbrace{\frac{g^2}{2}} |H_1 \bar{H}_2|^2$$

gauge couplings, in contrast to SM

Five physical states: $h^0, H^0, A^0, H^\pm$ (no $\mathcal{CP}$V at tree-level)

2 $\mathcal{CP}$-violating phases: ξ, arg(m_{12})

Input parameters: $\tan\beta = \frac{v_2}{v_1}$, $M_{H^\pm}$

Contrary to the SM:

m_h is not a free parameter

MSSM tree-level bound: $m_h < M_Z$, excluded by LEP Higgs searches

Large radiative corrections:

Dominant one-loop corrections:

$$\Delta m_h^2 \sim G_\mu m_t^4 \ln\left(\frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2}\right)$$

> The MSSM Higgs sector is connected to all other sector via loop corrections (especially to the scalar top sector)

Measurement of m_h, Higgs couplings $\Rightarrow$ test of the theory

LHC: $\Delta m_h \approx 0.2$ GeV

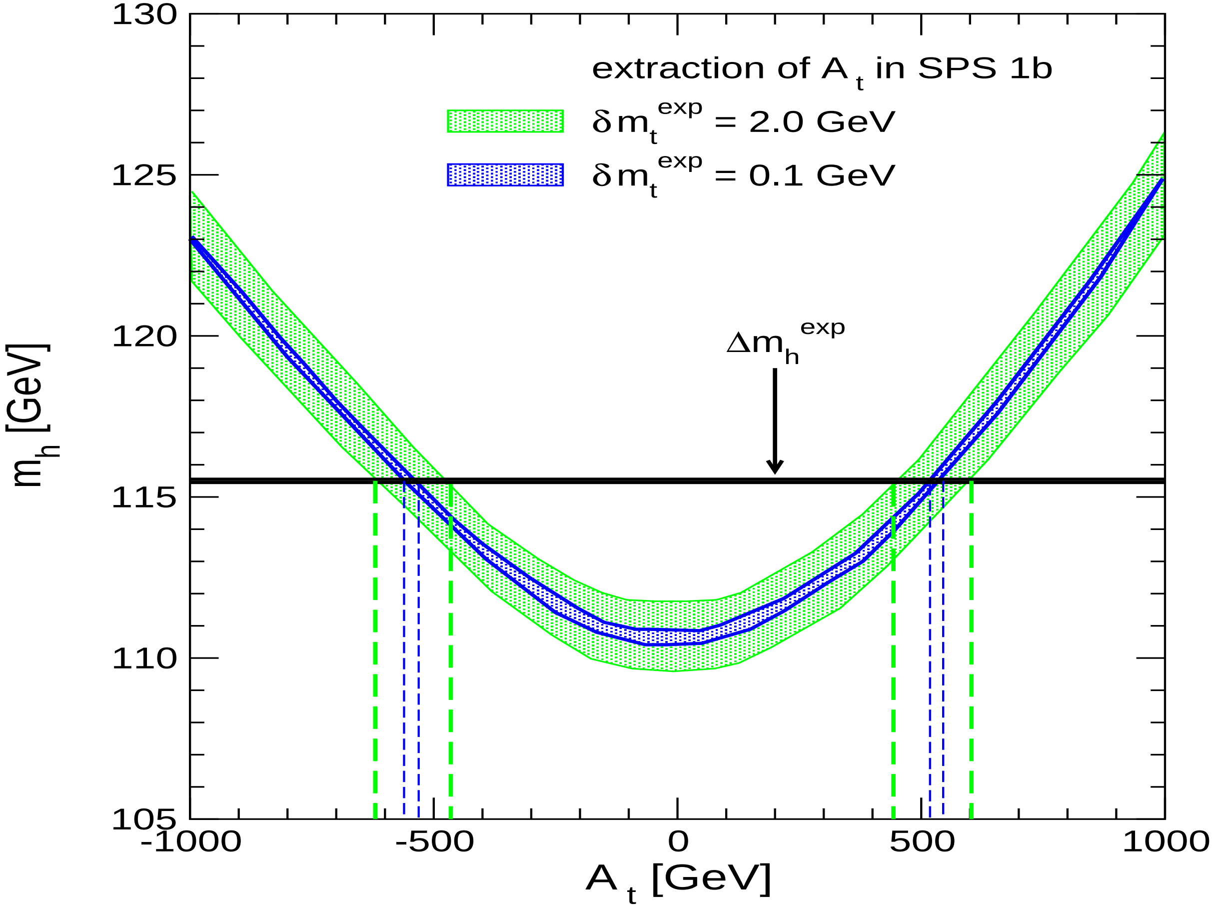
ILC: $\Delta m_h \approx 0.05$ GeV

$\Rightarrow$ aim for theoretical precision!

($\Rightarrow m_h$ will be (the best?) electroweak precision observable)

Example of application/motivation (I): m_h prediction as a function of A_t

[*S.H., S. Kraml, W. Porod, G. Weiglein '02*]



SPS1b:

$m_{\tilde{t}_1}, m_{\tilde{t}_2}, m_{\tilde{b}_1}, m_{\tilde{b}_2}$ known,

A_t unknown

$\tan\beta, M_A$ known,

realistic parametric errors assumed

(from SUSY exp. errors)

$\Rightarrow$ extraction of A_t possible

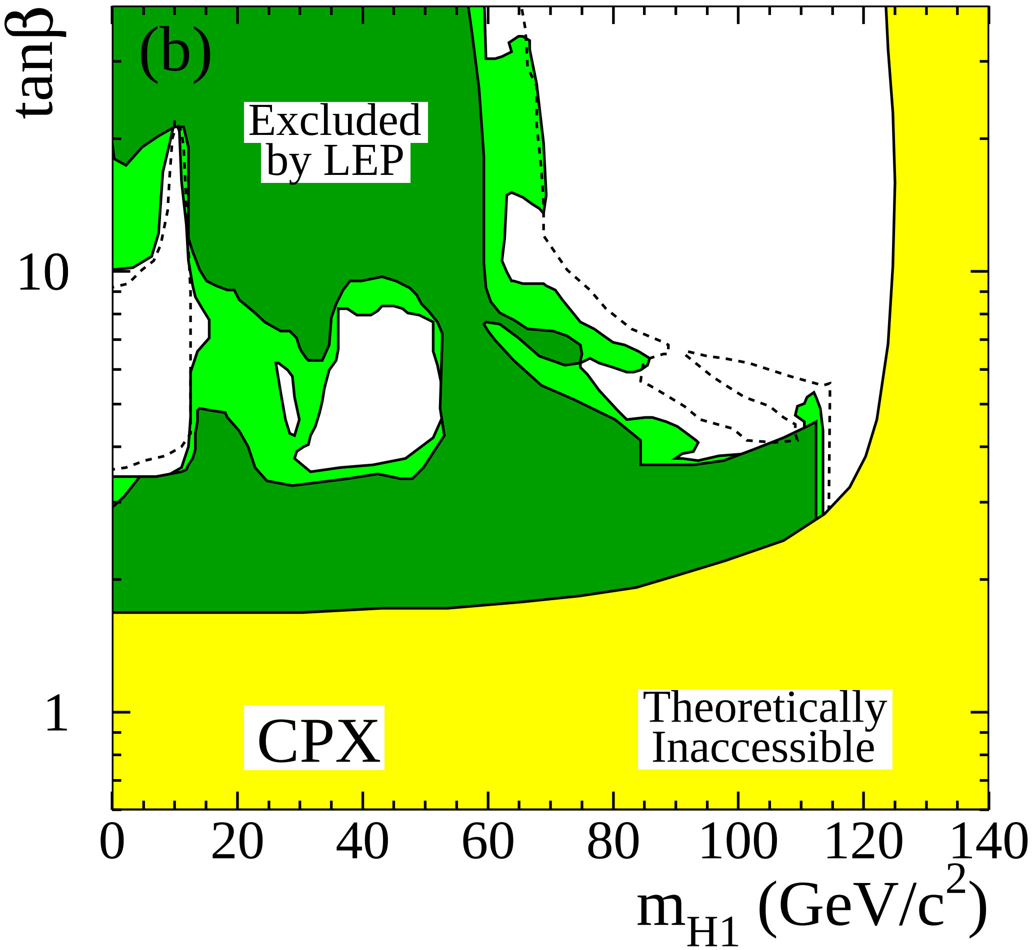
Theory error neglected

$\Rightarrow$ m_h is crucial input for SUSY fit programs (Fittino, Sfitter)

Example of application/motivation (II):

search for cMSSM Higgs bosons at LEP

[*LEP Higgs Working Group '06*]



$\Rightarrow$ Large unexcluded domains

even for very low Higgs masses

$\Rightarrow$ precise prediction for Higgs masses and decay widths necessary

The Higgs sector of the cMSSM at tree-level:

- phase of m_{12} :

  $m_{12} = 0$ and $\mu = 0 \Rightarrow$ additional $U(1)$ (PQ) symmetry

  reality: $m_{12} \neq 0$, $\mu \neq 0$

  $\Rightarrow$ perform PQ transformation with ϕ_{PQ}

$$
\begin{aligned}
m_{12}^{2\prime} &= |m_{12}^2| e^{i(\phi_{m_{12}} - \phi_{\mathrm{PQ}})} \\
\mu' &= |\mu| e^{i(\phi_\mu - \phi_{\mathrm{PQ}})}
\end{aligned}
$$

  $\Rightarrow$ m_{12} can always be chosen real

- phase of H_2: ξ :

  mixing between $\mathcal{CP}$-even and $\mathcal{CP}$-odd states:

$$
\mathcal{M}_{\mathcal{CP}-\text{even},\mathcal{CP}-\text{odd}} = \begin{pmatrix} 0 & m_{12}^2 \sin\xi \\ -m_{12}^2 \sin\xi & 0 \end{pmatrix}
$$

  Tadpoles have to vanish: $T_A^{\text{tree}} \propto \sin\xi \, m_{12}^2 \stackrel{!}{=} 0$

  $\Rightarrow \xi = 0 \Rightarrow$ no $\mathcal{CP}$V at tree-level

The Higgs sector of the cMSSM at the loop-level:

Complex parameters enter via loop corrections:

– μ : Higgsino mass parameter

– $A_{t,b,\tau}$: trilinear couplings $\Rightarrow$ $X_{t,b,\tau} = A_{t,b,\tau} - \mu^*\{\cot\beta\,, \tan\beta\}$ complex

– $M_{1,2}$: gaugino mass parameter (one phase can be eliminated)

– M_3 : gluino mass parameter

$\Rightarrow$ can induce $\mathcal{CP}$-violating effects

Result:

$$(A, H, h) \to (h_3, h_2, h_1)$$

with

$$m_{h_3} > m_{h_2} > m_{h_1}$$

$\Rightarrow$ strong changes in Higgs couplings to SM gauge bosons and fermions

$\tilde{t}/\tilde{b}$ sector of the MSSM: (scalar partner of the top/bottom quark)

Stop, sbottom mass matrices ($X_t = A_t - \mu^*/\tan\beta$, $X_b = A_b - \mu^* \tan\beta$):

$$\mathcal{M}^2_{\tilde{t}} = \begin{pmatrix} M^2_{\tilde{t}_L} + m_t^2 + DT_{t_1} & m_t X_t^* \\ m_t X_t & M^2_{\tilde{t}_R} + m_t^2 + DT_{t_2} \end{pmatrix} \xrightarrow{\theta_{\tilde{t}}} \begin{pmatrix} m^2_{\tilde{t}_1} & 0 \\ 0 & m^2_{\tilde{t}_2} \end{pmatrix}$$

$$\mathcal{M}^2_{\tilde{b}} = \begin{pmatrix} M^2_{\tilde{b}_L} + m_b^2 + DT_{b_1} & m_b X_b^* \\ m_b X_b & M^2_{\tilde{b}_R} + m_b^2 + DT_{b_2} \end{pmatrix} \xrightarrow{\theta_{\tilde{b}}} \begin{pmatrix} m^2_{\tilde{b}_1} & 0 \\ 0 & m^2_{\tilde{b}_2} \end{pmatrix}$$

mixing important in stop sector (also in sbottom sector for large $\tan\beta$)

soft SUSY-breaking parameters A_t, A_b also appear in ϕ-$\tilde{t}/\tilde{b}$ couplings

$$m^2_{\tilde{t}_{1,2}} = m_t^2 + \frac{1}{2}\left(M^2_{\tilde{t}_L} + M^2_{\tilde{t}_R} \mp \sqrt{(M^2_{\tilde{t}_L} - M^2_{\tilde{t}_R})^2 + 4m_t^2 |X_t|^2}\right)$$

$\Rightarrow$ independent of ϕ_{X_t}

but $\theta_{\tilde{t}}$ is now complex

$SU(2)$ relation $\Rightarrow M_{\tilde{t}_L} = M_{\tilde{b}_L} \Rightarrow$ relation between $m_{\tilde{t}_1}, m_{\tilde{t}_2}, \theta_{\tilde{t}}, m_{\tilde{b}_1}, m_{\tilde{b}_2}, \theta_{\tilde{b}}$

2. Two-loop corrections in the cMSSM Higgs sector

Inclusion of higher-order corrections:

$\rightarrow$ Feynman-diagrammatic approach

Propagator / mass matrix with higher-order corrections:

$$\begin{pmatrix} q^2 - M_A^2 + \hat{\Sigma}_{AA}(q^2) & \hat{\Sigma}_{AH}(q^2) & \hat{\Sigma}_{Ah}(q^2) \\ \hat{\Sigma}_{HA}(q^2) & q^2 - m_H^2 + \hat{\Sigma}_{HH}(q^2) & \hat{\Sigma}_{Hh}(q^2) \\ \hat{\Sigma}_{hA}(q^2) & \hat{\Sigma}_{hH}(q^2) & q^2 - m_h^2 + \hat{\Sigma}_{hh}(q^2) \end{pmatrix}$$

$\hat{\Sigma}_{ij}(q^2)$ $(i,j = h, H, A)$: renormalized Higgs self-energies

$\hat{\Sigma}_{Ah}, \hat{\Sigma}_{AH} \neq 0 \Rightarrow \mathcal{CPV}$, $\mathcal{CP}$-even and $\mathcal{CP}$-odd fields can mix

Our result for $\hat{\Sigma}_{ij}$:

– full 1-loop evaluation: dependence on all possible phases included

– New: $\mathcal{O}(\alpha_t\alpha_s)$ corrections in the FD approach
  rMSSM: difference between FD and RGiEP approach $\mathcal{O}$(few GeV)

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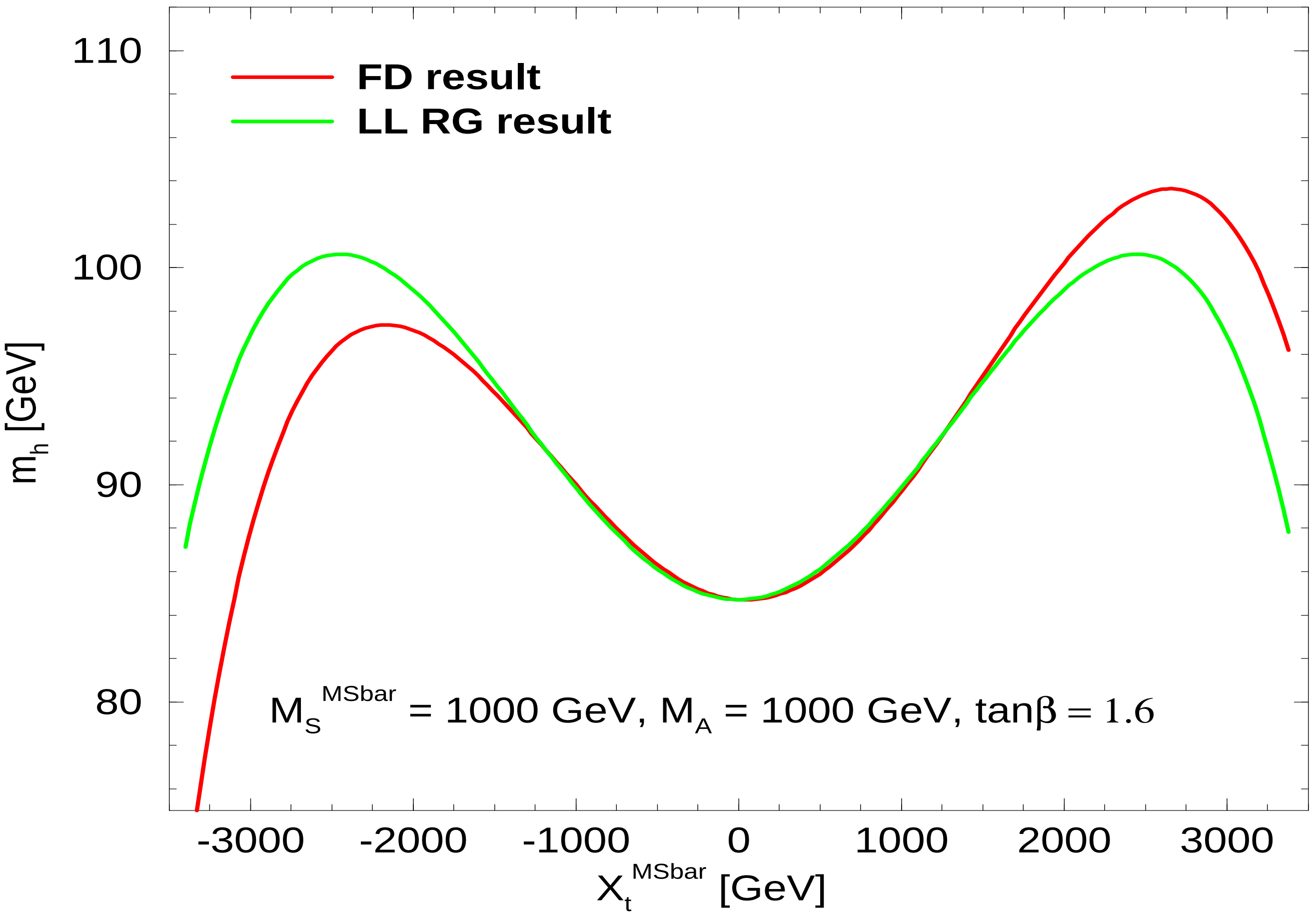
[*M. Carena, H. Haber, S.H., W. Hollik, C. Wagner, G. Weiglein '00*]



⇒ same order of differences expected for the complex case

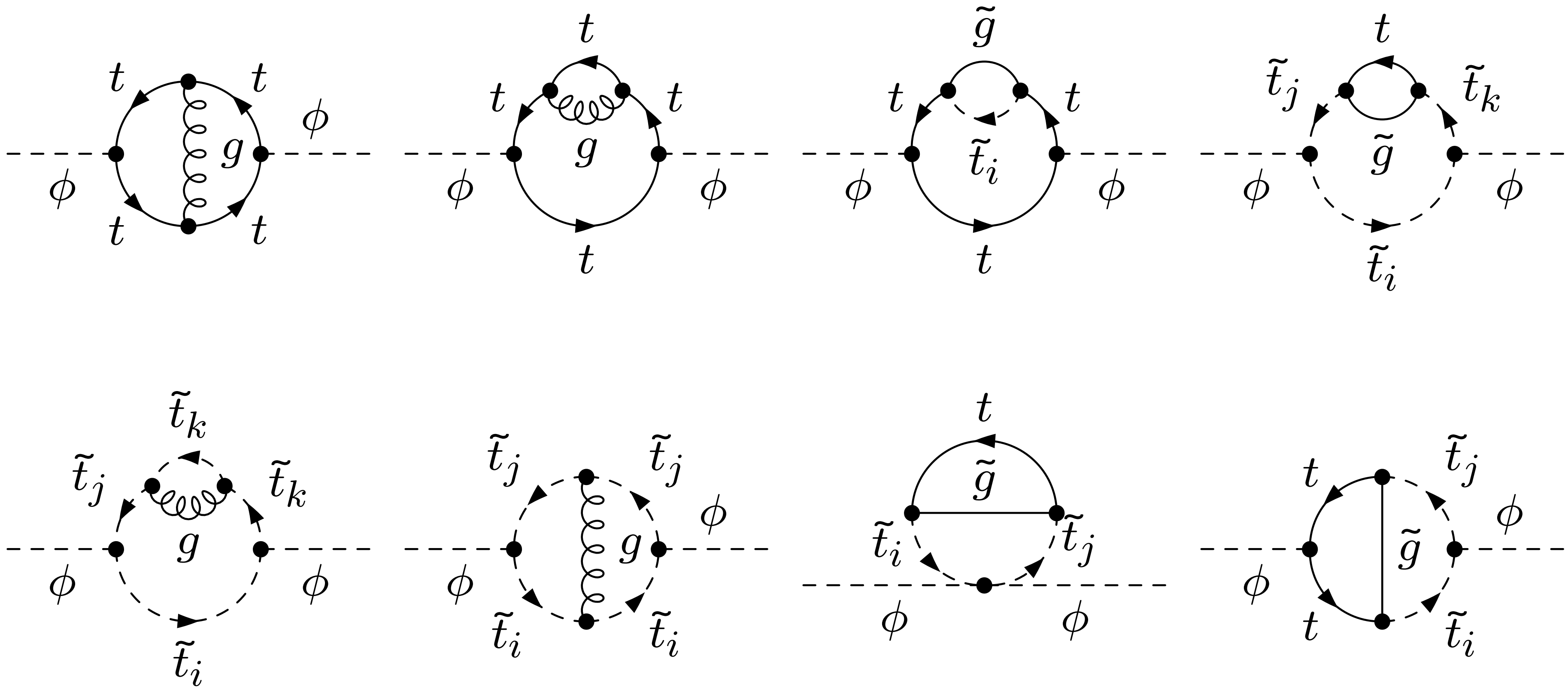
$\mathcal{O}(\alpha_t\alpha_s)$ corrections in the FD approach

- only y_t^2 contributions
- $g, g' \to 0$
- external momentum $\to 0$

$\Rightarrow$ Two-loop diagrams $\to$ T

Contributions to the 2-loop self-energy:

2-loop self-energy diagrams:



$\phi = h, H, A$

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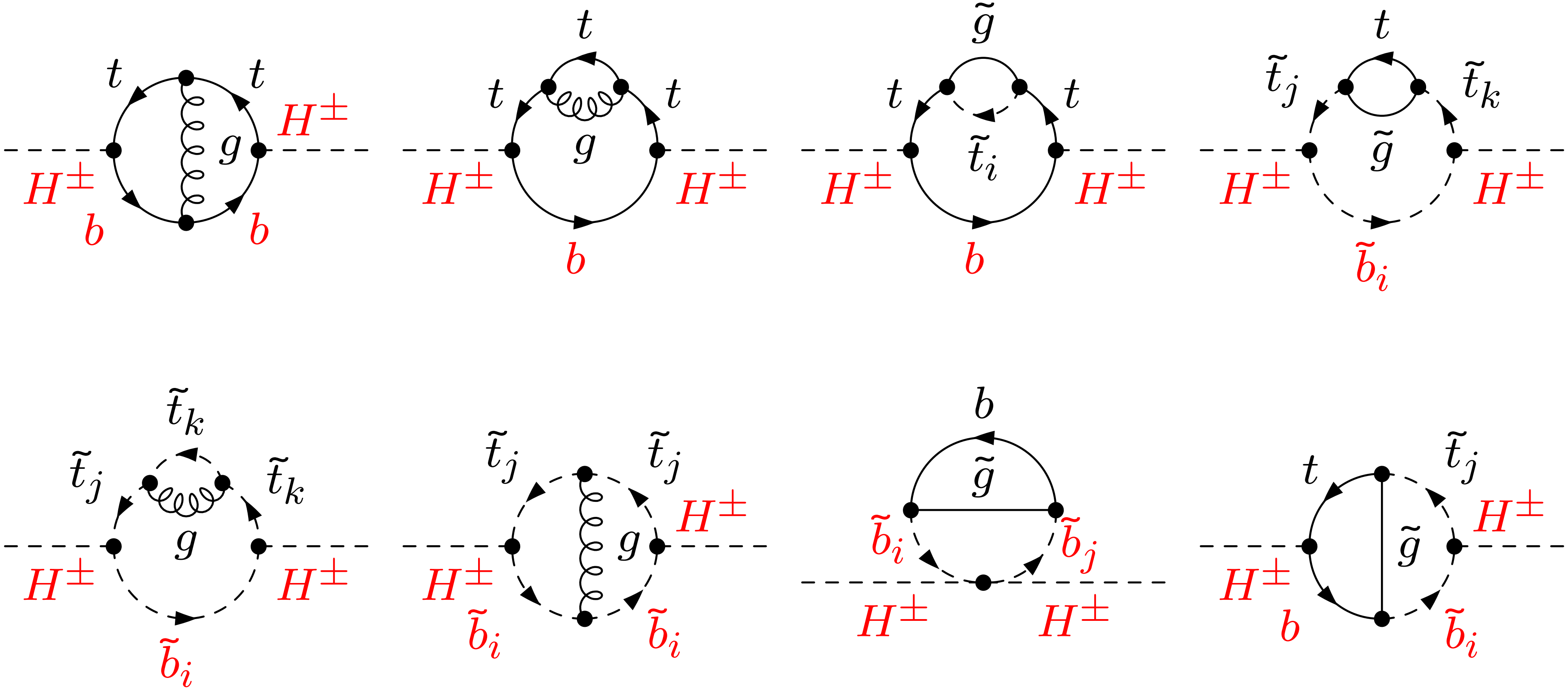

new: $H^\pm$ as external Higgs (via renormalization)

$\Rightarrow b/\tilde{b}$ enter (even diagrams without $t/\tilde{t}$: $H^+H^-\tilde{b}_i\tilde{b}_j \sim y_t^2$)

$\mathcal{O}(\alpha_t \alpha_s)$ corrections in the FD approach

- only y_t^2 contributions
- $g, g' \to 0$
- external momentum $\to 0$

$\Rightarrow$ Two-loop diagrams

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$\mathcal{O}(\alpha_t\alpha_s)$ corrections in the FD approach

– only y_t^2 contributions

– $g, g' \to 0$

– external momentum $\to 0$

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new: $H^{\pm}$ as external Higgs (via renormalization)
$\Rightarrow b/\tilde{b}$ enter (even diagrams without $t/\tilde{t}$)

Differences to real case:

– $\Rightarrow b/\tilde{b}$ enter

– A_t complex $\Rightarrow$ complex $\tilde{t}$ mixing angle enters

– M_3 complex , but $m_{\tilde{g}}$ is real (and positive)
$\Rightarrow$ phase of M_3 enters gluino vertices

$\Rightarrow$ many more scales

$\Rightarrow$ Renormalization . . .

Evaluation of 2-loop diagrams:

1. Generation of diagrams and amplitudes with FeynArts

   [*Küblbeck, Böhm, Denner '90*] [*Hahn '00 - '05*]

2. Algebraic evaluation and tensor integral reduction to scalar integrals: TwoCalc

   (works for two-loop self-energies)

   [*G. Weiglein '92*] [*G. Weiglein, R. Scharf, M. Böhm '94*]

3. Further evaluation: insertion of integrals, expansion in $\delta = \frac{1}{2}(4 - D)$
   $\rightarrow$ algebraical check: cancellation of divergencies

4. Result:

   – algebraic Mathematica code
   – Fortran code (currently implemented into *FeynHiggs*)

$\mathcal{O}(\alpha_t\alpha_s)$ corrections in the FD approach: Renormalization (I)

Two-loop renormalization:

$$\hat{\Sigma}^{(2)}_{hh} = \Sigma^{(2)}_{hh} + c^2_{\beta-\alpha}\delta M^{2\,(2)}_{H^\pm} + \frac{e}{2M_Z s_W c_W}\left(c_{\beta-\alpha}s^2_{\beta-\alpha}\delta T^{(2)}_H - s_{\beta-\alpha}(1+c^2_{\beta-\alpha})\delta T^{(2)}_h\right)$$

$$\hat{\Sigma}^{(2)}_{hA} = \Sigma^{(2)}_{hA} - \frac{e}{2M_Z s_W c_W} s_{\beta-\alpha}\delta T^{(2)}_A$$

- use $M_{H^\pm}$ as on-shell mass, since A mixes with h, H in higher orders
  $\Rightarrow$ $\tilde{b}$ sector enters via $\delta M^2_{H^\pm} = \Sigma_{H^\pm}$
  $\Rightarrow$ renormalization of the $\tilde{b}$ sector $\longrightarrow$ subloop renormalization

- loop corrections: $T_A \neq 0 \Rightarrow$ renormalized to zero
  $\Rightarrow \delta T_A = T_A$ enters renormalized self-energies $\hat{\Sigma}_{hA}$, $\hat{\Sigma}_{HA}$

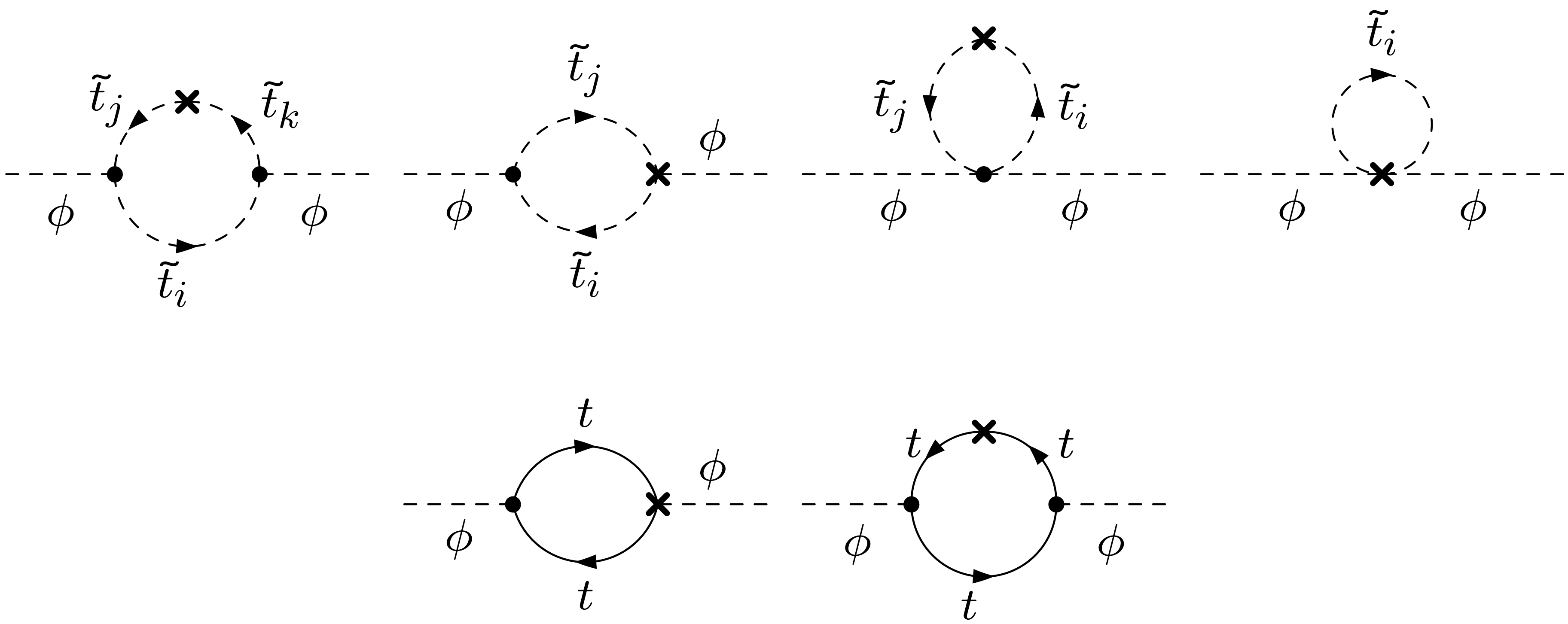
$\mathcal{O}(\alpha_t\alpha_s)$ corrections in the FD approach: Renormalization (II)

sub-loop renormalization:

$\Rightarrow$ One-loop diagrams with CT insertion $\longrightarrow$ T

Contributions to the 2-loop self-energy:

diagrams with counter term insertion:



$\phi = h, H, A$

Contributions to the 2-loop self-energy:

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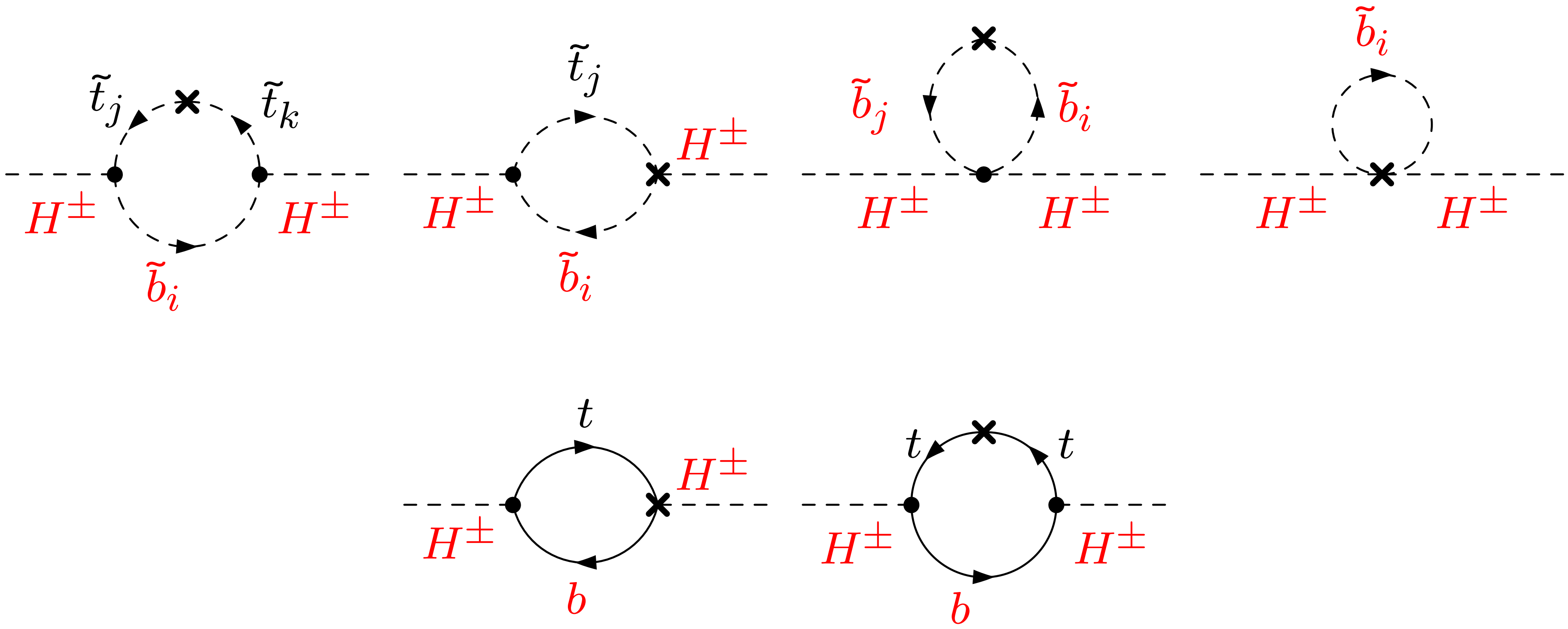


new: $H^\pm$ as external Higgs
$\Rightarrow b/\tilde{b}$ enter (even diagrams without $t/\tilde{t}$)
$\Rightarrow$ renormalization of the $\tilde{b}$ sector

$\mathcal{O}(\alpha_t\alpha_s)$ corrections in the FD approach: Renormalization (II)

sub-loop renormalization:

$\Rightarrow$ One-loop diagrams with CT insertion

- $\tilde{b}$ sector enters via $\delta M^2_{H^\pm} = \Sigma_{H^\pm}$

  $\Rightarrow$ renormalization of the $\tilde{b}$ sector: $m_b = 0 \Rightarrow$ only $\delta m_{\tilde{b}_L}$

- 1) A_t complex

  $\Rightarrow$ renormalization of $|A_t|$ and ϕ_{A_t}: $\delta A_t = e^{i\,\phi_{A_t}}\delta|A_t| \; + \; i\,A_t\,\delta\phi_{A_t}$

  (no renormalization of μ, no $\mathcal{O}(\alpha_s)$ corrections)

  $\Rightarrow$ $\overline{\text{DR}}$ renormalization

- 2) $\theta_{\tilde{t}}$ complex

  $\Rightarrow$ renormalization of $|\theta_{\tilde{t}}|$ and $\phi_{\tilde{t}}$:

  $\Rightarrow$ On-shell renormalization via

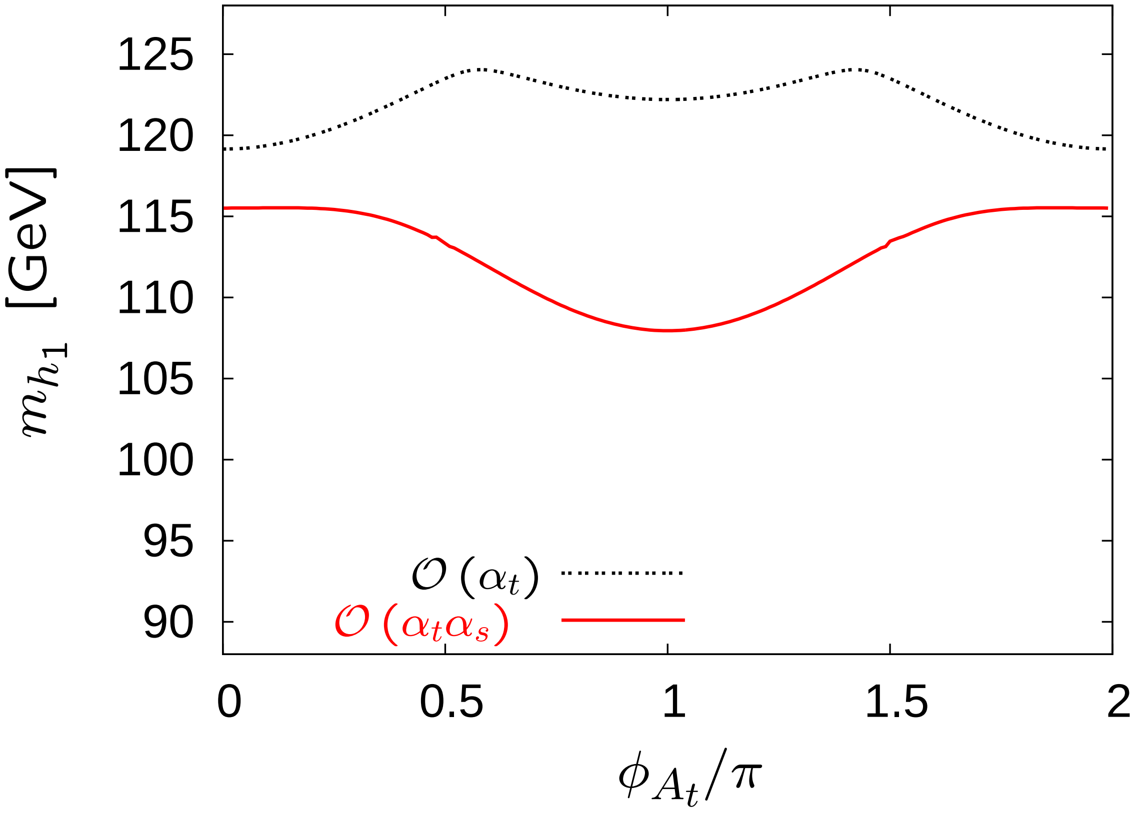
$$\widetilde{\text{Re}}\,\hat{\Sigma}_{\tilde{t}_{12}}(m^2_{\tilde{t}_1}) + \widetilde{\text{Re}}\,\hat{\Sigma}_{\tilde{t}_{12}}(m^2_{\tilde{t}_2}) \;\stackrel{!}{=}\; 0$$

$$\Rightarrow \widetilde{\text{Re}}\,\Sigma_{\tilde{t}_{12}}(m^2_{\tilde{t}_1}) + \widetilde{\text{Re}}\,\Sigma_{\tilde{t}_{12}}(m^2_{\tilde{t}_2}) \;=\; e^{i\phi_{\tilde{t}}}(m^2_{\tilde{t}_1} - m^2_{\tilde{t}_2}) \times (\delta\theta_{\tilde{t}} + i\,s_{\tilde{t}}\,c_{\tilde{t}}\,\delta\phi_{\tilde{t}})$$

  $\Rightarrow$ evaluate $\delta|A_t|$ and $\delta\phi_{A_t}$ as dependent parameters

3. Numerical results

m_{h_1} as a function of ϕ_{A_t}:

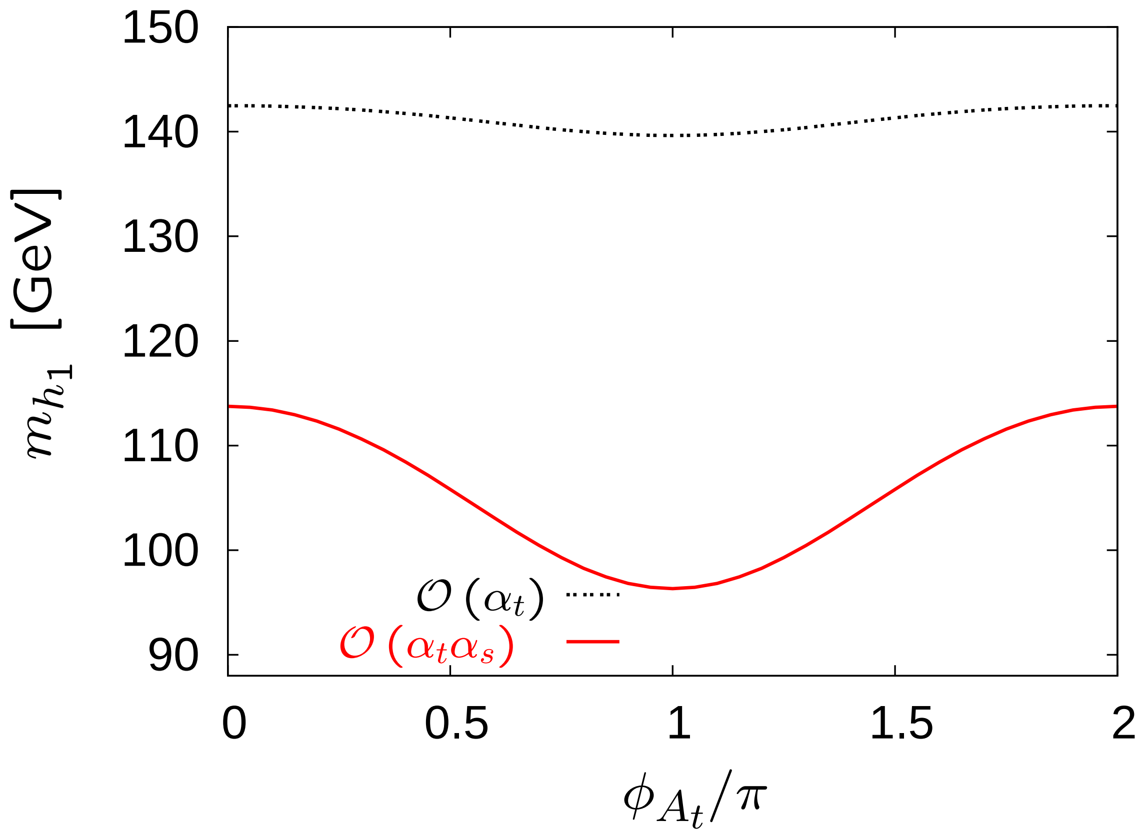


$M_{\text{SUSY}} = 1000$ GeV
$|A_t| = 2000$ GeV
$\tan\beta = 10$
$M_{H^\pm} = 150$ GeV

OS renormalization

$\Rightarrow$ modified dependence
on ϕ_{A_t} at the 2-loop level

m_{h_1} as a function of ϕ_{A_t}:

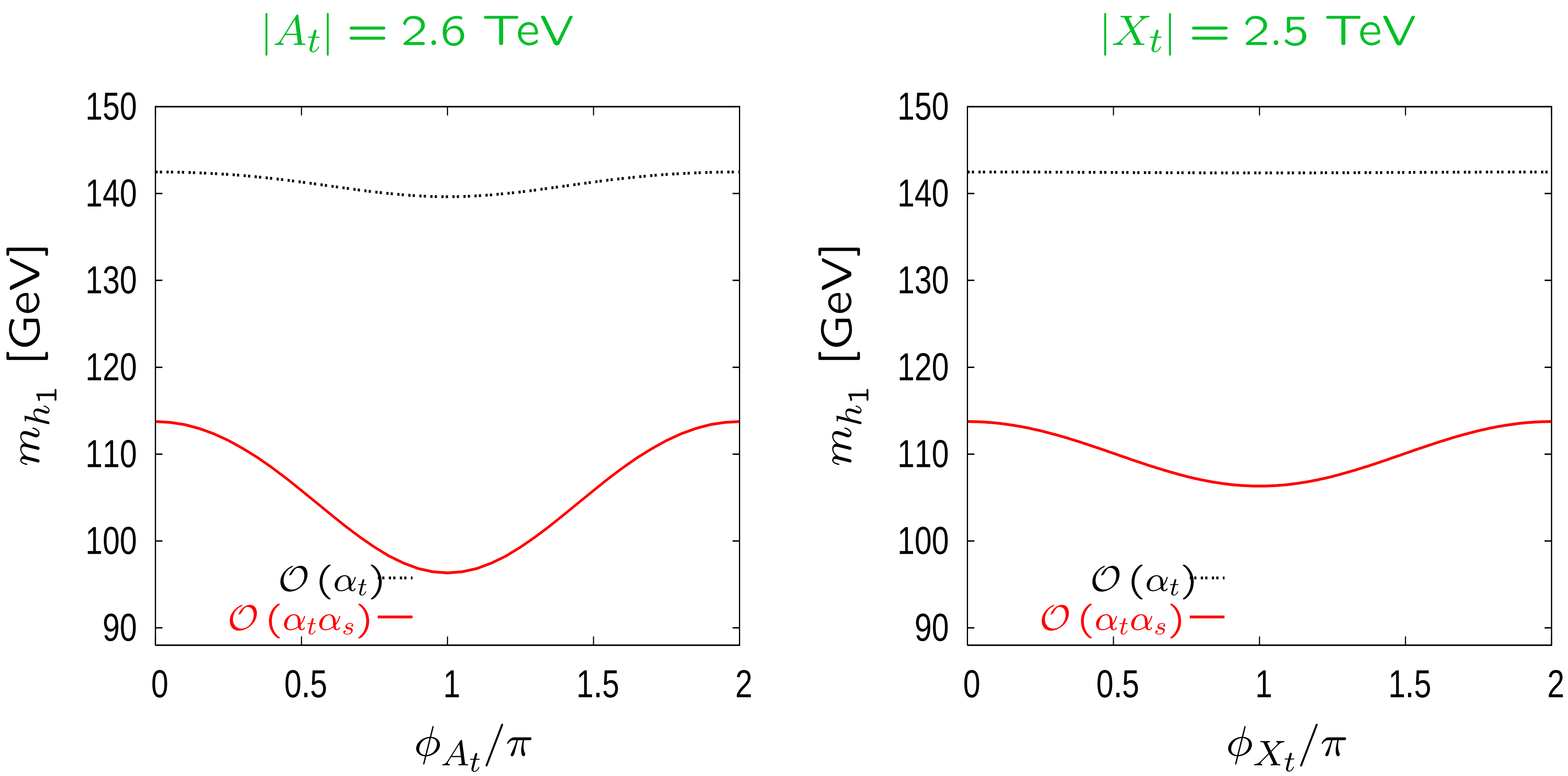


$M_{\text{SUSY}} = 1000$ GeV
$|A_t| = 2000$ GeV
$\tan\beta = 10$
$M_{H^\pm} = 500$ GeV

OS renormalization

$\Rightarrow$ modified dependence
on ϕ_{A_t} at the 2-loop level

Comparison of ϕ_{A_t} and ϕ_{X_t} dependence:

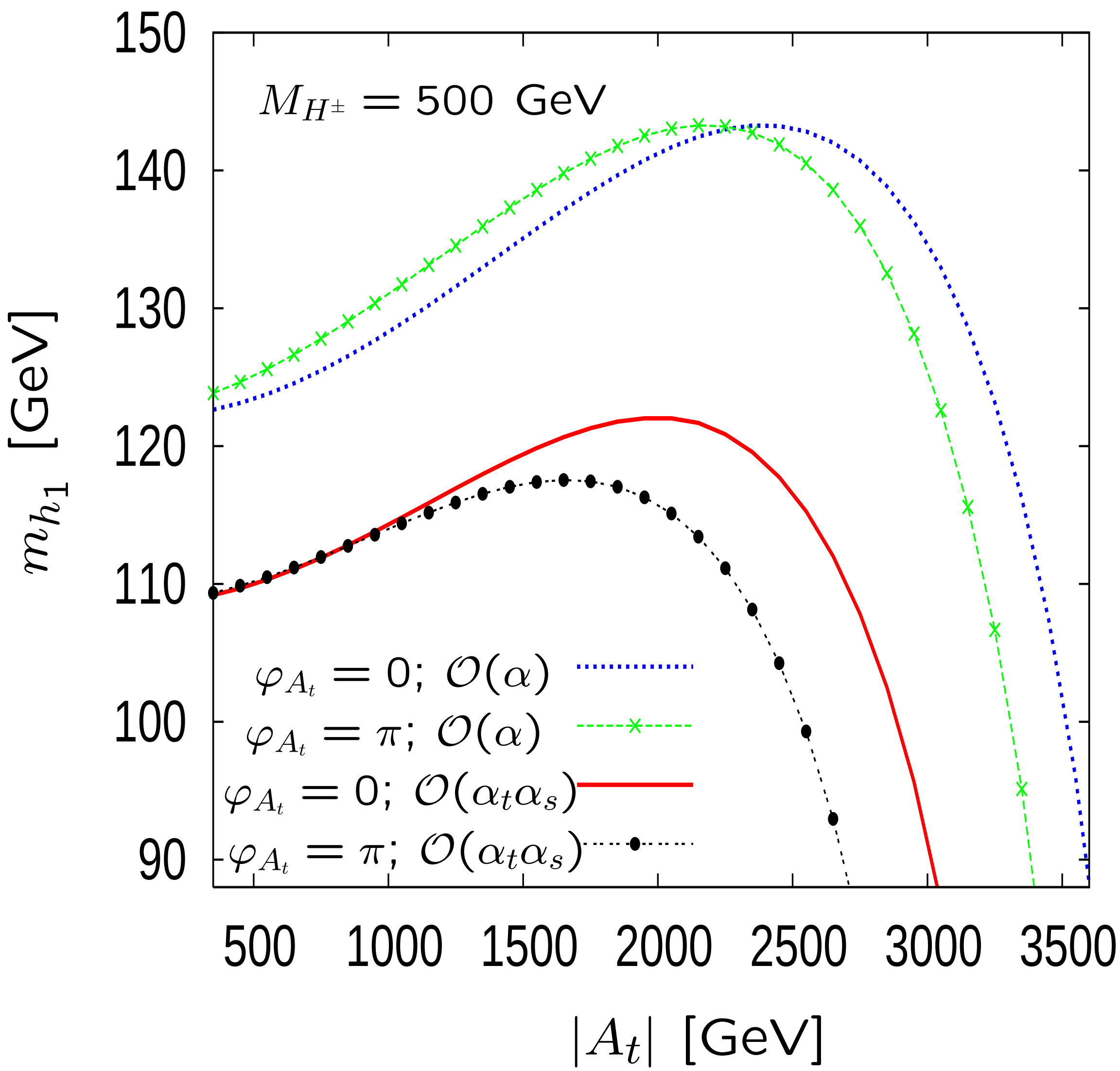


⇒ one-loop: phase dependence only via shift in $\tilde{t}$ masses

⇒ two-loop: addition, real phase dependence

(smaller than full ϕ_{A_t} dependence)

m_{h_1} as a function of A_t:



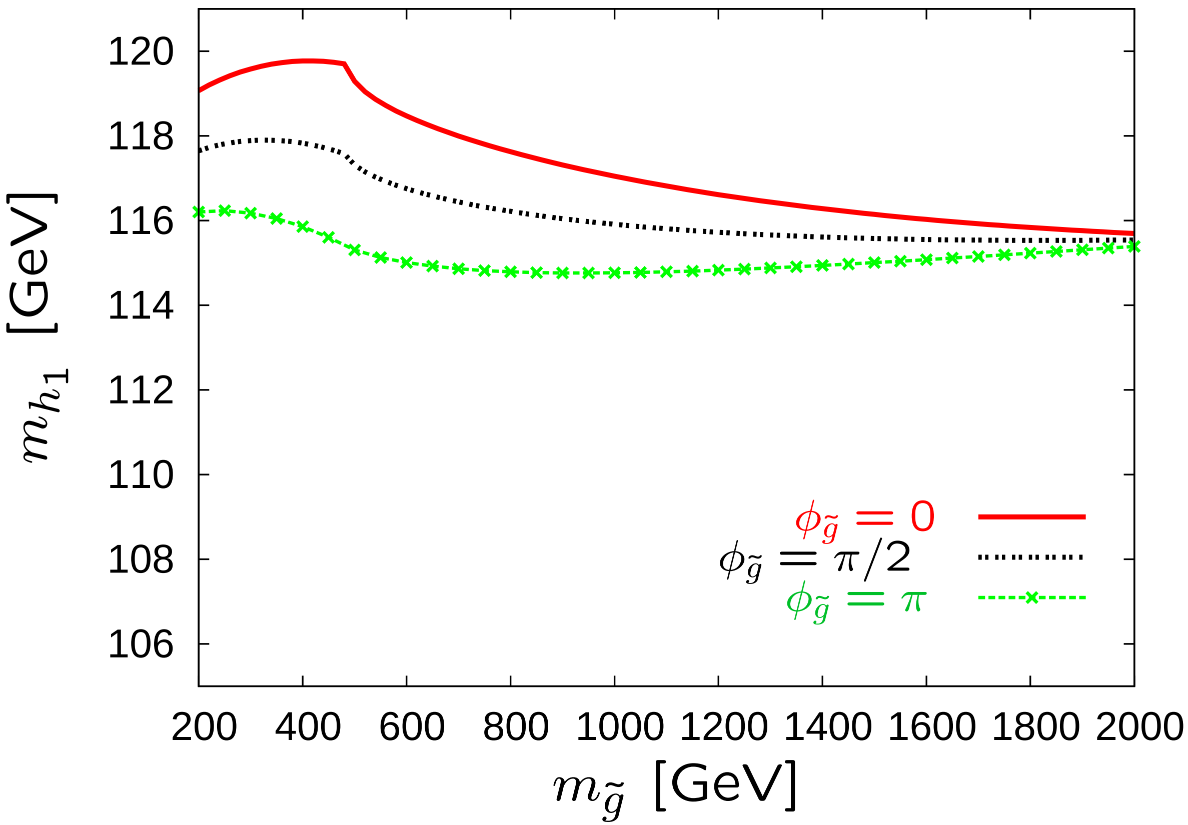
$M_{\text{SUSY}} = 1000$ GeV
$|A_t| = 2000$ GeV
$\tan\beta = 10$
$M_{H^\pm} = 500$ GeV

OS renormalization

1L: shift in $|A_t|$,
no change in max[m_{h_1}]

2L: max[m_{h_1}] depends
on ϕ_{A_t}
position of maximum
shifted

m_{h_1} as a function of $\phi_{\tilde{g}}$:



$M_{\text{SUSY}} = 500$ GeV
$A_t = 1000$ GeV
$\tan\beta = 10$
$M_{H^\pm} = 500$ GeV

OS renormalization

$\Rightarrow$ threshold at $m_{\tilde{g}} = m_{\tilde{t}} + m_t$

$\Rightarrow$ large effects around threshold

$\Rightarrow$ phase dependence has to be taken into account

4. Conclusinos

- – The LHC/ILC will provide high precision results for a light Higgs
  – MSSM Higgs masses and couplings is connected via radiative corrections to all other sectors
  $\Rightarrow$ good motivation for high-precision (two-loop) calculation

- Evaluation of $\mathcal{O}(\alpha_t\alpha_s)$ corrections in the cMSSM:

  $\Rightarrow$ two-loop self-energies (vanishing ext. momentum) with many scales
  – new: $\Sigma_{H^\pm}$ enters via renormalization $\Rightarrow$ $\tilde{b}$ sector enters
  – new: $\tilde{t}$ mixing complex
  – new: $\tilde{g}$ mass parameter complex
  – new: renormalization for complex parameters

- Numerical results:

  – large shifts in $|A_t|$ possible
  – ϕ_{A_t} dependence modified at the two-loop level
  – $\phi_{\tilde{g}}$ dependence $\mathcal{O}(2\text{ GeV})$ possible

- Results are currently implemented into *FeynHiggs* (www.feynhiggs.de)